Earth Physics and Global Glacial Isostasy: From Paleo-Geodesy to Space-Geodesy

W.R. Peltier
Department of Physics
University of Toronto, Toronto, Ontario, Canada, M5S 1A7

Abstract. The global phenomenon of glacial isostatic adjustment is one that involves internal Earth System interactions that are recorded in ways that are essentially "geodetic". This phenomenon, in which changes in the shape of the viscoelastic earth are driven by large scale exchanges of mass between the cryosphere and the oceans, is observable both "paleo-geodetically" and space-geodetically. Examples of paleo-geodetic measurements include ^{14}C dated histories of the changing level of the sea (the geoid) with respect to the surface of the solid earth, as well as measurements of the history of variations in the rate of axial rotation based upon the timing of ancient eclipses of the sun and moon. Examples of space geodetic measurements include joint VLBI, SLR and GPS observations of displacement rate vectors of "points" fixed to the surface of the solid earth, as well as satellite derived measures of the "anomalous" gravitational field. Observations of the time dependence of the gravity field that will be delivered by the GRACE mission are expected to further transform our ability to infer both earth structural properties (e.g. viscosity) and past and present degrees of cryospheric (in)stability. This brief paper reviews the present state of the theory that has been developed in order to exploit the totality of such geodetic observations so as to enable accurate geophysical inferences to be made based upon them.

1 Introduction

The development of the modern linear viscoelastic theory of the glacial isostatic adjustment process dates from a paper (Peltier 1974) that is now more than twenty five years old in which it was demonstrated how one might accurately compute the impulse response of a spherically symmetric and linearly viscoelastic model of the earth to surface mass load excitation. This theory built upon the formalism for the equivalent elastic problem that continues to serve as basis for our understanding of elastic load tides (Farrell, 1972). In the limit of linear viscoelasticity, it was shown that Laplace transform methods could be employed such that one could directly compute complex frequency spectra for the h, k and ℓ Love numbers in terms of which solutions for the various responses to impulsive surface loading could be constructed (Peltier 1976). These spectra may be inverted into the time domain using either the collocation method employed in Peltier (1974) or using the more accurate normal mode methodology invoked in Peltier (1976, 1985) a part of which had earlier been explored in Wu (1978) but not implemented numerically due to numerical instability problems that proved insurmountable at that time. The key impulse response function that is required for accurate computation of the relative sea level response to variations in surface ice load is that for the perturbation to the gravitational potential on the deforming surface of the solid earth. This space-time Green function was first computed in Peltier and Andrews (1976). With it, the stage was set for the accurate computation of relative sea level histories on the surface of a continuously deforming viscoelastic model of the Earth.

The methodology that was developed to perform such analyses built upon the earlier comments of Platzman (e.g. 1971) on the influence upon the ocean tides of the elastic yielding of the sea floor (see his remarks on page 266). A very primitive form of the integral equation that one must solve (to which I refer as the Sea Level Equation, or SLE) was presented in Farrell and Clark (1976), an integral equation for which the kernel was just the Green function for the gravitational potential perturbation computed in Peltier and Andrews (1976). The form of this theory was indeed primitive, however, as it was not formulated in a way that incorporated other than a point load representation of glaciation history. A first approximation to the northern hemisphere component of this history called ICE-1 was compiled in Peltier and Andrews (1976) and this was employed in both Clark, Farrell and Peltier (1978) and Peltier, Farrell and Clark (1978) in order to produce the first realistic predictions of relative sea level histories. These results had to be seen as preliminary, however, as the model of the radial viscoelastic structure employed by them did not include a lithosphere, the deglaciation history included no southern hemisphere component of the ice load, and the method employed to solve the

International Association of Geodesy Symposia, Vol. 123
Sideris (ed.), Gravity, Geoid, and Geodynamics 2000

SLE was primitive, being based upon a crude discretization of the interacting surface ice and ocean loads. At this time there also existed no global data base of ^{14}C dated relative sea level histories against which model predictions could be verified.

These drawbacks were no impediment, however, to early investigation of the issue of the rotational response of the earth to the glaciation-deglaciation process, initial results for which were reported in Sabadini and Peltier (1981). As pointed out in Peltier (1982) and Wu and Peltier 1984, however, the theory for the polar motion component of the rotational response presented in Sabadini and Peltier (1981) was seriously flawed as was the theory for this effect presented earlier by Nakiboglu and Lambeck (1980). The crux of this problem was as follows. Munk and McDonald (1960) had earlier demonstrated, using a viscoelastic model if the Earth that supported only a single mode of "viscous gravitational relaxation" (terminology from Peltier 1976), that at a time by which the surface load had become stationary, the net forcing of the polar wander vanishes and this component of the rotational response is thereby nullified. As demonstrated in Peltier (1982) and Wu and Peltier (1984), however, realistic models of the radial viscoelastic structure support more than a single mode of viscous gravitational relaxation and in this circumstance polar wander does not cease once the surface load becomes stationary. It was demonstrated through these analyses that the viscosity of the mantle that was required to enable the theory to fit both of the earth rotation observations was in close accord with that required to fit relative sea level histories.

In order to bring the totality of the observational data to bear on the formal inverse problems for mantle viscosity and deglaciation history, however, it was first necessary to compile a global data base of relative sea level histories and to produce an improved global model of the deglaciation process from last glacial maximum to present. Both these goals were achieved in Tushingham and Peltier (1991, 1992) who produced both a large compilation of ^{14}C dated rsl histories of "reconnaisance" quality as well as a much improved global model of the last deglaciation event of the current ice-age. This new model of late glacial ice thickness variations was called ICE-3G, a model which, although it has since been considerably improved, is still employed by many of the large number of scientists who are now active in this still growing field of activity. Tushingham and Peltier also produced a very simple 4-layer (one-dimensionl) model of the internal mantle viscosity structure which they called VM1 which appeared to adequately reconcile a significant quantity of data related to the isostatic adjustment process.

With the completion of the work of Tushingham and Peltier (1991, 1992) on the compilation of much improved data sets for glaciation history and for relative sea level observations, the stage was set for transition from the era of paleo-geodesy to the era of space-geodesy. In the next section of this review, I will briefly recapitulate the enhanced version of the theory that has been developed to enable close contact with space-geodetic measurements to be achieved. In the final section I'll comment upon the further advances that can be expected over the next several years as effort in this area continues.

2 Theoretical Glacial Isostasy in the Era of Space-Geodesy

If the viscoelastic structure of the interior of the earth may be considered to be spherically symmetric and the viscoelasticity to be describable in terms of a linear (say Maxwell) approximation, then the integral SLE takes the general form (e.g. see Peltier 1998a for a detailed discussion):

$$S(\theta,\lambda,t) = \left[\int_{--}^{t} dt^- \iint_{\lambda} d\lambda^- \left\{ L \sim G_\lambda^L - T \sim G_\lambda^T \right\} - \frac{\lambda\ (t)}{g} \right] \sim C(\theta,\lambda,t) \qquad (1)$$

in which C(θ, λ, t) is the ocean function which is unity over the ocean covered portion of the Earth's surface and zero elsewhere, L(θ, λ, t) is the surface mass load per unit area which has the composite form:

$$L(\theta,\lambda,t) = \lambda_I I(\theta,\lambda,t) - \lambda_w S(\theta,\lambda,t) \qquad (2)$$

in which λ_I and λ_w are the densities of ice and water respectively, I is the history of ice-sheet thickness variations and S is the same relative sea level history as appears on the left hand side of (1). T is the change in the centrifugal potential due to the changes in the rotational state of the planet induced by the glaciation-deglaciation process. The kernels in the integral equation (1) are Green functions for the surface load induced perturbation to the gravitational potential (G_λ^L) and for the centrifugal potential induced perturbation (G_λ^T). The remaining function λ (t) in (1) is determined in the process of constructing solutions so as to ensure that the surface load forcing conserves mass.

The complete algorithm that is currently employed to solve (1) has recently been reviewed in detail in Peltier (1998a). It is based upon the recognition that the impacts of both time variations in the ocean function and the changing rotation are relatively small. The algorithm employed to incorporate the former influence was first described in Peltier (1994) and makes use of the fact that (1), being a construct of first order perturbation theory, delivers solutions S that are relative to an undefined and therefore arbitrary datum. This fact may be exploited to define what I have termed "topographically self-consistent" solutions to the SLE that incorporate the influence of changes in the ocean function. As demonstrated in Peltier (1998b), when large changes in C occur, due to the transformation of initially ice-covered regions into ocean covered regions (Hudson Bay, Gulf of Bothnia, Barents and Kara Seas), the effect is non-perturbative such as to render the explicit ice load "I" employed in fitting the solutions of (1) to the observations, an effective load. The true ice-load may be inferred by an exact calculation of the "implicit ice" that must also have been removed using the theory of adjustment detailed in Peltier (1998b). With the additional correction to the solution S due to the changing rotational state constructed using the solution for the rotational response due to Peltier (1982) and Wu and Peltier (1984) and the modification to the centrifugal potential T, thereafter constructed using the results of Dahlen (1976), the algorithm for solution of the extended SLE (1) is complete.

Since the method of incorporating the influence of C(t) and T(t) is iterative, the methodology that is employed to solve the unperturbed SLE is clearly also important. For this purpose I continue to employ the semi-spectral algorithm described in detail in Mitrovica and Peltier (1991) in which the model fields S and I are currently represented on a basis of spherical harmonics that is rhomboidally truncated at degree and order 512. This high spatial resolution is required in order to accurately represent the evolving position of the coastline. Since the model fields are now described on such a high resolution basis of spherical harmonics, the methodology that is employed to refine the ice-thickness distribution in order to remove misfits to the observations that are unambiguously related to this cause has been significantly modified to exploit this fact. Whereas Tushingham and Peltier (1991), in the process of constructing ICE-3G, continued to employ the circular disk load representation of Peltier and Andrews (1976), a method which is subject to significant error as the two-sphere cannot thereby be tiled uniformly, in the construction of the ICE-4G model Peltier (1994) employed a different methodology. The ice load was specified in terms of the thickness history in each of a large number of predefined spherical parallelepipeds whose edges are defined by parallels of latitude and lines of longitude. Give the high spectral resolution employed in the representation of the fields the individual cells are extremely well resolved, the result being a much improved representation of the ice load. Because of the manner in which the ICE-3G model was discretized, users need to be aware of the errors that one is bound to incur when using it. This model should no longer be employed for GIA analyses. The more recently developed ICE-4G model is available for distribution and may now be obtained by contacting me via peltier@atmosp.physics.utoronto.ca.

Based upon the use of this more precise modelling structure the recent past has witnessed a number of advances in our ability to connect the predictions of this theory to modern geodetic observations. Especially important in this regard has been the implementation of methods to accurately compute both radial and tangential displacement rate vectors. Given that the model fields are now defined on a basis of spherical harmonics it is sensible to evaluate the convolution integrals in terms of which the displacement rate vector is defined using the spectral representations developed in Peltier (1976). The expressions for the radial displacement vector U(θ, λ, t) $\underline{r}$ and the tangential displacement vector $\underline{V}$ are simply as follows (see also Peltier 1995):

$$U(\theta,\lambda,t)=\sum_{\ell=o}^{\sim}\sum_{m=-\ell}^{-\ell}\left[\frac{4\lambda a^3}{(2\ell-1)\,m_e}\left(L_{\ell m}h_\ell^{E}-\sum_{k=1}^{k(\ell)}q_k^{\ell}\lambda_{\ell m}^{k}\right)\right]Y_{\ell m} \qquad (3a)$$

$$\underline{V}(\theta,\lambda,t)=\sum_{\ell=o}^{\sim}(t)\sum_{m=-\ell}^{-\ell}\left[\frac{4\lambda a^3}{(2\ell-1)\,m_e}\left(L_{\ell m}\ell_\ell^{E}-\sum_{k=1}^{k(\ell)}\lambda_k^{\ell}\lambda_{\ell m}^{k}\right)\right]\underset{\sim}{-}Y_{\ell m} \qquad (3b)$$

where

$$\lambda_{\ell m}^{k}(t)=\int_{--}^{t}L_{\ell m}(t^{-})\,e^{-s_k^{\ell}(t-t^{-})}\,dt^{-} \qquad (3c)$$

The remaining parameters in (3) are those which appear in the viscoelastic normal mode expansions of the h and ℓ Love numbers as:

$$h_\ell=h_\ell^{E}\lambda(t)-\sum_{k=1}^{K(\ell)}q_k^{\ell}e^{-s_k^{\ell}t} \qquad (4a)$$

$$\ell_\ell=\ell_\ell^{E}\lambda(t)-\sum_{k=1}^{K^1(\ell)}r_k^{\ell}e^{-s_k^{\ell}t} \qquad (4b)$$

The important vector $\underset{\sim}{Y}_{\ell m}$ that appears in (3b) is evaluated using the formula in Forte and Peltier (1994). This spectral structure has been employed to make predictions of displacement rate vectors in Mitrovica et al. (1994) and in Peltier (1995). Argus (1996) employed the vertical displacement results from the latter paper to investigate the issue as to whether VLBI data could be employed to directly observe the expected vertical motion and concluded that the existing array of radio-telescopes was observing rates of radial motion that agreed quite well with the predictions of the model of glacial isostatic adjustment. These analyses were recently extended by Argus et al. (1999) who intercompared the predictions of ICE-4G (VM1) and ICE-4G (VM2) against a set of combined VLBI and SLR observations. Their conclusions were that the VM2 viscosity model was indeed preferred on the basis of the vertical motion observations but that the horizontal motions seemed to prefer VM1. As pointed out recently elsewhere, the problem with the horizontal motions may well be associated with the ice load rather than with the viscosity profile VM2. Further analyses are currently underway by Argus et al. to fully incorporate the global array of GPS observations into the mix of data that is employed to verify the model. It is hoped that it will eventually be possible to access the GPS data from the BIFROST (1996) project for this purpose as well as those from the internationally accessible network.

In the present space-geodetic era it is expected that the most important next opportunity for advancing our understanding of the glacial isostatic adjustment process will follow from the measurements of the time dependence of the global gravity field that will be forthcoming from the GRACE mission. The measurement of time dependent geoid height will allow us to make direct contact with the global rate of sea level rise that is currently ongoing in the Earth system. As demonstrated in Peltier (1998a) these observations will be significantly influenced by the glacial isostatic adjustment process, not only over the continental regions that were previously glaciated but also over the global oceans. Over previously ice covered regions signals of order +1 mm/yr are expected whereas over the equatorial and higher latitude oceans the signal strength may reach a level near -0.4 mm yr^{-1}. Since the latter is a very significant fraction of the globally averaged rate of rsl rise of ~ 1.8 mm yr^{-1} that has been inferred on the basis of a set of tide gauge records that have been corrected for the glacial isostatic adjustment effect, it is quite clear that GRACE data will have to be similarly corrected in order to properly estimate the component of this global signal that is connected to modern climate change. Figure 1 shows the prediction of the ICE-4G (VM2) model of the GIA effect in terms of time dependent geoid height, both at global scale (a) and for four different localized regions, respectively North America (b), Fennoscandia (c), Antarctica (d) and eastern Eurasia (e). Inspection of these results demonstrates that the signal strengths to be expected over the formerly glaciated regions due to the GIA effect are on the order of +1.1 mm yr^{-1} over N. America and Antarctica and 0.4 mm/yr over Fennoscandia. Over the equatorial oceans, however, where absolute sea level is falling due to this cause, the signal strength is of order -0.4 mm yr^{-1}. These signals should be detectable with the GRACE measurement system.

4 Conclusions

Although the spherically symmetric, linearly viscoelastic, model of the glacial isostatic adjustment process that I have continued to develop in Toronto has successfully explained a very wide variety of observations connected to the glacial isostatic adjustment process, a number of issues remain that will require further analysis. Firstly, it will be clear on physical grounds that the effective viscosity of the mantle of the earth cannot be spherically symmetric. Not only is the thickness of the effectively elastic lithosphere well known to be a strong function of geographic position, but the viscosity in the sublithospheric region is also laterally heterogeneous. I expect that a non-perturbative method will be required to properly calculate the impact of such effects upon the solution. At present, however, there appear to be few systematic misfits of the theory to the observations that would require one to employ models involving such significantly higher levels of physical and numerical complexity. As the quantity and quality of the data we have at our disposal increase, however, this circumstance may change.

Of perhaps more fundamental interest is the question as to whether mantle rheology is truly Newtonian as assumed in the Maxwell model or whether the stress relaxation process that controls glacial isostasy is simply controlled by rheological transience. Arguments have been recently presented on both sides of this important question (Pari and Peltier 1995, Peltier 1996, Forte and Mitrovica 1996, Pari and Peltier 2000, Butler and Peltier 2000). Based upon the most recent analyses of Butler and Peltier, the convection timescale viscosity must be significantly higher than that which governs the rebound process.

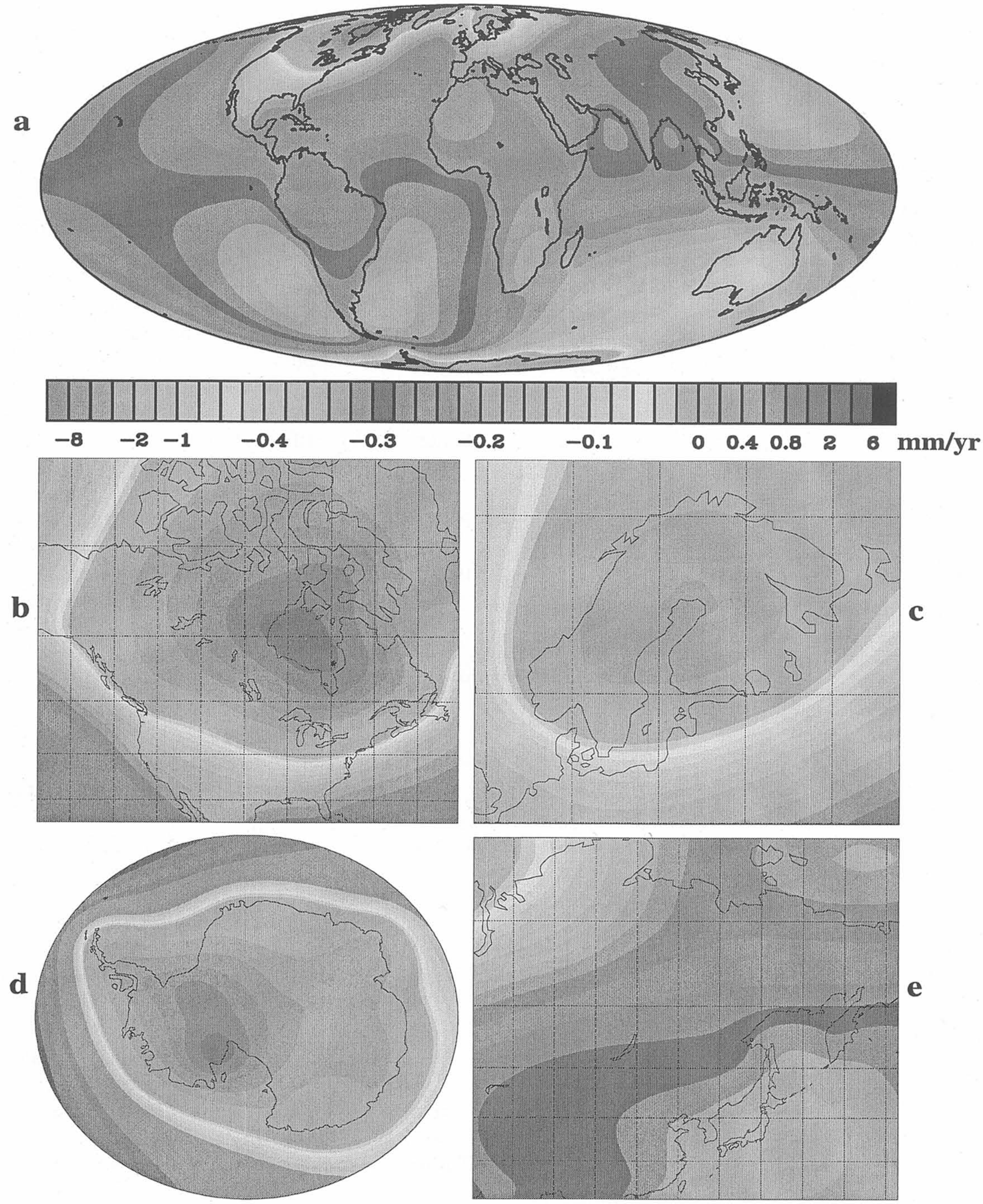

Fig. 1 (a) The predicted present day rate of change of geoid height, shown in Mollweide projection obtained by solving equation (1) in the text using the ICE-4G deglaciation model in the VM2 viscosity model as inputs. The full influence of rotational feedback is included. The remaining parts of the Figure show "blow-ups" of the global field shown in part (a) for the previously glaciated regions of (b) North America and (c) North Western Europe, for (c) the partly deglaciated region of Antarctica and for (e) the new significantly glaciated region of Asia.

References

Argus, D.F., Postglacial rebound from VLBI geodesy: On establishing vertical reference, *Geophys. Res. Lett.*, 23, 973-976, 1996.

Argus, D.F., W.R. Peltier and M.Watkins, Glacial isostatic adjustment observed using very long baseline interferometry and satellite laser ranging geodesy, *J. Geophys. Res.*, 10, 29077-29093, 1999.

BIFROST Project Members, GPS measurements to constrain geodynamic processes in Fennoscandia, *EOS Trans.*, AGU, 77, 337, 341, 1996.

Butler, S. and W.R. Peltier, On scaling relations i time dependent mantle convection and the heat transfer constraint on layering, *J. Geophys. Res.*, 105, 3175-3208, 2000.

Clark, J.A., W.E. Farrell, and W.R. Peltier, Global changes in postglacial sea level: A numerical calculation, *Quat. Res.*, 9, 265-287, 1978.

Dahlen, F.A., The passive influence of the oceans upon the rotation of the Earth, *Geophys. J.R. Astron. Soc.*, 46, 363-406-1976.

Farrell, W.E., Deformation of the Earth by surface loads, *Rev. Geophys.*, 10, 761-797, 1972.

Farrell, W.E., and J.A. Clark, On postglacial sea level, *Geophys. J.R. Astron. Soc.*, 46, 647-667, 1976.

Forte, A.M., and J.X. Mitrovica, New inferences of mantle viscosity from joint inversion of long-wavelength mantle convection and post-glacial rebound data, *Geophys. Res. Lett.*, 23, 1147-1150, 1996.

Forte, A.M., and W.R. Peltier, The kinematics and dynamics of poloidal-toroidal coupling in mantle flow: The importance of surface plates and lateral viscosity variations, *Adv. Geophys.*, 36, 1-119, 1994.

Mitrovica, J.X., and W.R. Peltier, On postglacial geoid subsidence over the equatorial oceans, *J. Geophys. Res.*, 96, 20,053-20,071, 1991.

Mitrovica, J.X., J.L. Davis, and I.I.. Shapiro, A spectral formalism for computing three dimensional deformations due to surface loads, 2, Present-day glacial isostatic adjustment, *J. Geophys. Res.*, 99, 7075-7101, 1994.

Munk, W.H., and G.F. MacDonald, The *Rotation of the Earth* Cambridge Univ. Press, New York, 1960.

Nakiboglu, S.M. and K. Lambeck, Deglaciation effects upon the rotation of the Earth, *Geophys. J.R. astro. Soc.*, 62, 49-58.

Pari, G., and W.R. Peltier, The heat flow constraint on mantle tomography-based convection models: Toward a geodynamically self-consistent inference of mantle viscosity, *J. Geophys. Res.*, 100, 12,731-12,751, 1995.

Peltier, W.R., The impulse response of a Maxwell Earth, *Rev. Geopphys.*, 12, 649-669, 1974.

Peltier, W.R., Glacial isostatic adjustment, II, The inverse problem, *Geophys. J.R. Astron. Soc.*, 46, 669-706, 1976.

Peltier, W.R., Dynamics of the ice-age Earth, *Adv. Geophys.*, 24, 1-146, 1982.

Peltier, W.R., The LAGEOS constraint on deep mantle viscosity: Results from a new normal mode method for the inversion of viscoelastic relaxation spectra, *J. Geophys. Res.*, 90, 9411-9421, 1985.

Peltier, W.R., Ice age paleotopography, *Science*, 265, 195-201, 1994.

Peltier, W.R., VLBI baselines from the ICE-4G model of postglacial rebound, *Geophys. Res. Lett.*, 22, 465-468, 1995.

Peltier, W.R., Mantle viscosity and ice-age ice sheet topography, *Science*, 273, 1359-1364, 1996.

Peltier, W.R., Postglacial variations in the level of the sea: implications for climate dynamics and solid earth geophysics, *Rev. Geophys.*, 36, 603-684, 1998a.

Peltier, W.R., "Implicit ice" in the global theory of glacial isostatic adjustment, *Geophys. Res. Lett.*, 25, No. 21, 3955-3958, 1998b.

Peltier, W.R., and J.T. Andrews, Glacial isostatic adjustment, I, The forward problem, *Geophys. J.R. Astron. Soc.*, 46, 605-646, 1976.

Peltier, W.R., W.E. Farrell, and J.A. Clark, Glacial isostasy and relative sea level: A global finite element model, *Tectonophysics*, 50, 81-110, 1978.

Sabadini, R. and W.R. Peltier, Pleistocene deglaciation and the Earth's rotation: implications for mantle viscosity, *Geophys. J.R. astr. Soc.*, 66, 553-578, 1981.

Tushingham, A.M., and W.R. Peltier, ICE-3G: A new global model of late Pleistocene deglaciation based upon geophysical predictions of post-glacial relative sea level change, *J. Geophys. Res.*, 96, 4497-4523, 1991.

Tushingham, A.M. and W.R. Peltier, Validation of the ICE-3G model of Wurm-Wisconsin deglaciation using a global data base of relative sea level histories, *J. Geophys. Res.*, 97, 3285-3304, 1992.

Wu, P., The response of a Maxwell Earth to applied surface loads: Glacial isostatic adjustment, M.Sc. thesis, Dept. of Phys., Univ. of Toronto, Toronto, Ontario, Canada, 1978.

Wu, P., and W.R. Peltier, Pleistocene deglaciation and the Earth's rotation: A new analysis, *Geophys. J.R. Astron. Soc.*, 76, 202-242, 1984.